
Nova Scotia
Practitioner Training and Certification Program
Nova Scotia Practitioner Training and Certification Program

Practitioner’s Handbook for the Math Modules:

Teaching Math: Basic Principles (MATH BP)

Teaching Math: Keeping It Real (MATH KIR)

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Truro, Nova Scotia
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This publication is available electronically on Literacy Nova Scotia’s website at www.ns.literacy.ca
Literacy Nova Scotia wishes to acknowledge and thank the following for their contributions:

1) The Office of Literacy and Essential Skills (HRSDC) for funding this project.
2) The Department of Labour and Workforce Development, Adult Education Division; Nova Scotia Community College; Nova Scotia Provincial Library; National Adult Literacy Database for their support and partnership.
3) The members of the Project Work Group who provided valuable input during the development of the program and the comments and feedback during revisions:
   - Allan Banks
   - Avril Lewis
   - Donna MacGillivray
   - Julie Nickerson
   - Catherine Wile
4) The members of the Project Advisory Group who provided valuable advice for the project and the comments and feedback to complete this report:
   - Ann Marie Downie - Literacy Nova Scotia
   - Marlene Duckworth – Queens County Learning Network
   - Peter Gillis - Valley Community Learning Association
   - Meredith Hutchings - Department of Labour and Workforce Development
   - Earl Letts – National Adult Literacy Database
   - Nova Scotia Provincial Library
   - David Pilmer- Department of Labour and Workforce Development
   - Mel Pothier - Nova Scotia Community College
   - Grail Sangster-Guysborough County Adult Learning Association
   - Lynne Wells-Orchard- Department of Labour and Workforce Development
5) Those who took part in the piloting of the program and provided valuable feedback: Matt Taylor, Andrea Manthorne, Patricia Cloutier, André Davey, Shannon Davis, Carolynne Nemecek, Hannah Mills, Kirsteen Thompson, Jennifer Steeves, Brenda Nemecek, Donna Casey, Pam O’Neil, Tracey Grosse, Jane Bolivar, Valerie Cheel, Pat Robinson, Sue Balkam, Dale Taylor, Bonnie Boivin, Gayle Morrison; as well as their network and regional Adult Education coordinators
6) The project team – Jayne Hunter, Program Manager; Kate Nonesuch, Curriculum Writer; Elaine Frampton, IT Specialist; and Gary Mason, Pilot Facilitator.

Thanks to David Pilmer for many contributions to this manual, especially the section on number sense.
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Introduction

This handbook will provide background reading as you work on both math modules of the Nova Scotia Practitioner Training Program, and will be a useful reference as you work with learners in math Levels I to III or preparing for the math portion of the GED.

The handbook will give some background on the learners in the Adult Learning Program, and then will focus on several elements of successful math teaching in community programs associated with the Nova Scotia School of Adult Learning (NSSAL). These elements are: providing explicit instruction; dealing with resistance; using real life materials; using manipulatives; teaching number sense and mental math; metacognition (learning about learning); encouraging automaticity; and making appropriate assessments. Each of these elements is important at each and every level of the ALP.

The remainder of the handbook focuses on teaching techniques, and gives detailed instructions for some proven methods of working with learners. Many additional resources for teaching math at these levels are available from the Instructor’s website of NSSAL (http://instructors.gonssal.ca/; username: adulted; password: NSSAL). More details about these resources are given in the sections that follow.

Basic Principles

Effective math instruction requires more than the old-fashioned methods which required the learner to do a lot of rote learning of methods of computation, and to apply that rote learning in carefully controlled pages of “word problems” which were really the same problem over and over with different numbers. Rather than watching the teacher do math, the learners need to do math themselves; rather than “drill and kill,” they need an opportunity to understand and think mathematically. Rather than insisting on one method and one answer, practitioners in the Adult Learning Program encourage diverse thinking about math problems, and recognize a variety of ways to go about finding the solution to a problem.

The following table is taken from the introduction to the Mathematics Level II curriculum, pages 9 – 10. The principles listed and the roles described have long been associated with instruction in communications, but less so with math instruction. Nevertheless, successful math instructors and tutors in the Adult Learning Program have embraced them alongside their colleagues who teach reading and writing.
## Characteristics of Effective ALP Delivery

<table>
<thead>
<tr>
<th>Principle of Adult Learning</th>
<th>Role of Instructors and Administrators</th>
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| Learning is a process of actively constructing meaning. | • create learning environments that foster investigation, debate, participation, exploration, communication, questioning, collecting, and finding ways to predict  
• provide learners with meaningful experiences  
• help learners to develop methods of learning |
| Learners construct knowledge and make it meaningful by relating it to their prior knowledge and experiences. | • find out what learners already know and can do  
• create learning environments and plan experiences that build on learners’ prior knowledge  
• acknowledge and respect learners’ learning experiences that may influence their ways of perceiving, thinking, feeling, and approaching the world  
• respect and support learners’ racial, cultural, and social identities  
• recognize, value, and use the great diversity of experiences and information learners bring with them  
• make sure that the learning materials reflect the diversity of learners  
• make sure learners are challenged to build on their prior knowledge, integrating new knowledge with what they already understand |
| Learning is enhanced when it takes place in a social and collaborative environment. | • make discussion, group work, and collaborative ventures central to classroom activities  
• structure opportunities where learners can interact in diverse social activities  
• make sure learners recognize the importance of transferring social and collaborative skills into their everyday lives  
• help learners to see themselves as members of a community of learners |
| Learners are able to see their learning as an integrated whole. | • plan opportunities to help learners make connections across the curriculum and link them with the outside world  
• provide learners with opportunities to apply strategies from across the curriculum to problems in real situations |
| Learners must see themselves as capable and successful. | • make sure that all learners experience genuine success on a regular basis  
• value experimentation and approximation as signs of growth  
• provide learning experiences and resources that reflect the diversity of the local and global community  
• provide learning opportunities that develop self-esteem without using self-esteem as a goal in itself |
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Principle of Adult Learning | Role of Instructors and Administrators
---|---
Learners have different ways of representing knowledge. | • recognize each learner’s preferred style of constructing meaning and provide opportunities for exploring alternative ways
• recognize, acknowledge, and build on learners’ diverse ways of representing knowledge—showing what they have learned
• plan a variety of open-ended experiences and assessment strategies

Reflection is an integral part of learning. | • observe and reflect on their own learning processes and experiences
• challenge their own beliefs and practices through continuous reflection
• encourage learners to observe and reflect on their own learning processes and experiences
• encourage learners to acknowledge and articulate their learning needs, styles, and preferences
• help learners use their reflections to change their behaviours and adjust their learning strategies


The Learners in the Adult Learning Program

Learners who come to take math at a community program have many different goals and reasons for enrolling, and their particular histories will provide you with information to chart a course for them to meet their needs. However, many of them have much in common. They all have done poorly in math at some point in their past; if they had done well, they wouldn’t be coming to you! Along with a history of not doing well in math comes a whole bag of emotions. Many of them will lack confidence in their ability to do math, or to think mathematically. Some will think of themselves as stupid, because of previous math failure, and many will say they must be lazy, assuming the reason for their failure was that they did not work hard enough, and perhaps having been called lazy because of math work undone or poorly done. Some will be angry because they want to be working or being trained for a job, but need to go through math with you in order to get the certificate that would get them into the job or the training. They may see math as an unnecessary barrier to reaching their goal.

Many of these learners have made repeated efforts formally and informally to learn mathematics. Some of them may have a learning disability specifically in mathematics, which was never recognized. Although the community-based organizations and the Department of Labour and Workforce Development currently do not know what percentage of our learners have a diagnosed or undiagnosed learning disability (LD), it is safe to assume, based on numbers from other adult learning programs in North America, that a significant number of our learners do have learning difficulties. The implication is
that traditional teaching techniques may not be best suited for all of these learners. For example, many learners still do not know their multiplication facts; this is not for lack of trying. Simply asking them to go and memorize a new set of facts each night or each week—the process they encountered in public school—would likely be unsuccessful. Teaching practices and materials that focus on multiplication strategies, rather than memorization of isolated facts, will better help these learners master their multiplication tables. Although instructors and tutors continually assess the progress of each learner and adapt teaching practices to meet the needs of the individual learner, this is of greater importance with those who have a possible LD in mathematics.

What moves an adult to enroll in a community-based program? There are many motives: to get their grade 12; to get a better job, or to keep the one they have; to qualify to enter a training program; to improve basic skills; to help their kids with school math; to be a role model to help kids stay in school; and so on. No one, however, comes to a program to learn math, or to learn to love math or because they want to think about math and learn more. They come because they want to get through it, to pass it, to get the piece of paper so they can get on with their goals. It is a tribute to the math instructors and tutors of community programs that many learners come to like math and to be interested in it for its own sake.

As mentioned, some learners want to move from community-based programs (typically Levels I and II) to the Nova Scotia Community College programs (typically Levels III and IV) so that they can ultimately receive a grade 12 diploma. Some learners do not wish to follow this path, and others are incapable of following it. (The Department of Labour and Workforce Development recognizes that designing curriculum that solely serves the needs of learners who wish to obtain a high school diploma is problematic. For this reason, the curriculum was designed with a greater emphasis on, and exploration of "life-skills" mathematics.) Others want to take the GED as a way of getting a certificate more quickly. A table that compares the GED with the ALP program is posted at http://www.gonssal.ca/General-Public/documents/GEDvs.Diploma.pdf

Learners who are also working on improving reading skills may find that their difficulties with reading make math more difficult; they may have trouble reading word problems, or understanding directions in texts or on tests. Here the math teacher becomes a reading teacher as well.

Learners come with many strengths, and often have well-developed strategies for doing the math they need in their daily life. Examples abound:

- Sam can add and subtract halves, quarters, eighths and sixteenths with no trouble, but couldn’t say what a common denominator is, and would have trouble adding 1/5 and 1/2.

- Mel makes complicated beading patterns based on the number of beads in the row she starts with, but doesn’t understand the terms “factor” and “multiple,” and doesn’t know most of her times tables.

- Del does accurate mental math in many situations, but can’t remember the steps to use pencil and paper to solve a question like this one: 248 – 159.
Although she doesn’t have much money, Maria is never embarrassed at the cash register by not having enough cash to pay for the groceries in her cart, but she can’t make a reasonable estimate of the answer to a problem in her math book.

So how do we teach math to people with such a variety of backgrounds and strengths? The sections that follow will help you work with learners to develop a math program that suits their interests, abilities and strengths, and leads to a successful learning outcome for them and for you.
Explicit Instruction

Many elementary and middle school math curriculums require students to investigate mathematical concepts through activities. This emphasis on “discovery learning,” asking adults to “play” enough to discover the relationships between numbers, and to invent methods of calculating with numbers, is problematic for adults for three reasons:

- Adults are not blank slates. They come with memories of how to do things (“put the 5 down here and carry the one over to here”) that interfere with a pure discovery method.
- Expediency is a factor for adult learners. They do not have several years to discover a handful of mathematical concepts.
- A significant number of our learners have diagnosed or undiagnosed learning disabilities in mathematics. Several research studies have shown that discovery learning can be problematic for low-achieving math students, and can severely compromise the learning of students with a math disability. They concluded that more scaffolded and/or explicit instruction better serves the learning needs of these students.

So we use explicit instruction, showing how to do something, modeling our own mathematical thinking. This does not mean that our teaching focuses on the mechanics of math, that is, the use of algorithms (step-by-step procedures for doing computations). If learners primarily spend their time mastering algorithms, they are often left with the misconception that mathematics is about memorizing procedures. Rather we want to go beyond “do it this way because that’s the way it’s done.” We want to make sure learners understand the math behind “do it this way.” When learners understand the meaning underlying a method of computing something, they can rely less on memory, which frees up more of their brain for thinking.
Resistance

Some learners throw themselves into school with such positive attitudes that there is no resistance; we only have to make sure that they have successful experiences in math to maintain their enthusiasm. Many learners, however, are less open to new strategies for learning math; their responses range from silent withdrawal, to questioning their value, to open refusal to use them.

Resistance can come in other forms. Some learners will attend regularly, outwardly exhibit a positive attitude, and complete a large number of “drill and kill” practice questions, yet fail to move on and learn new concepts or engage in higher order thinking around previously learned concepts. In most cases they lack confidence in their mathematical abilities and have become resistant to change. They are more than content to engage in busywork, and fail to see that they are making little progress.

In Changing the Way We Teach Math (2006), Kate Nonesuch writes about dealing with resistance:

I have developed a teaching stance that recognizes, honours, and encourages open expression of their resistance. As a result, many students will question my methods when they are new to them. …

Nearly every student who enrolls in a basic math class has years of (unsuccessful) experience as a math student; it stands to reason that they have a firm idea of what math class should be and what success in math looks like. They expect me to give them sheets of questions and some tricks to help them remember how to work with fractions. When I ask them to work with manipulatives or visuals, do group activities or field trips, they resist. “This is not real math.”

I deal with that resistance by acknowledging that what I am asking them to do is not what they are used to, and it feels strange. I ask them to tell me all the ways they have tried to learn math in the past. Then I ask, “Does anyone know a way to learn math that really works?” Invariably, nobody does because they have all been previously unsuccessful. This conversation with students is part of making my work and theory transparent, and makes them partners in designing their own learning. The discussion about past methods of learning math, an evaluation of what parts were more useful or less useful, and the conclusion that something new needs to be tried, means that they are part of the team talking about what form teaching will take (p.11).
Math in Real Life

We use relevant materials for several reasons, all to do with helping the learner make meaning, in this case, come to understand the mathematics involved in a particular situation or problem.

When a learner is learning something new in math class, it will increase understanding if you start from a real life situation where the new ideas come into play. For example, a learner who is learning about proportions will know that 2:6 is the same as 1:3 if she thinks about making orange juice, and will know that if she puts in more water than the recommended proportion, say 1:5, she will likely get complaints about the drink being too weak. And making the juice in both strengths will make the point stronger.

When a learner has developed an understanding of a mathematical idea, but needs to remember facts or procedures, it may help to associate the new information to daily life. For example, someone who is learning to add or subtract decimals may remember to line up the decimal points by remembering how lists of prices look, with the decimals lined up, rather than the initial digits.

Associating math ideas with daily life may also work in reverse—daily life may come to be associated with math. Someone who has talked about batting averages for years, without understanding the mathematical idea of “average” may come to understand the meaning in math class, but will be triggered to think about math when he is watching baseball. Whenever you trigger math thinking outside math class, that is a bonus!

Using relevant material allows learners to bring their own knowledge and experience of the situation to test the reasonableness of their mathematical computations. A learner who is asked to figure out how many hot dogs to cook for 7 people if the average person will eat 1.75 hot dogs may put the decimal in the wrong place and give an answer of 122.5. She can figure out that her answer is wrong by imagining 7 people for supper, and trying to fit 122 hot dogs into her cooking pot. On the other hand, if she is asked to figure out $7 \times 1.75$ without a context, and makes the same decimal point error, she has to rely solely on mathematical thinking to check the reasonableness of her answer.

The crux of the issue for numeracy practitioners is to make sure that the materials they are using are actually relevant. If you are a cook who uses recipes and measuring cups and spoons, you may find lots of math work in doubling recipes or cutting them in half, but those examples will be meaningless for learners who never bake, and who cook by judging the correct amount of ingredients by eye or by handfuls. Learners who do not have a bank account will not find banking examples relevant when they are trying to learn about adding or subtracting, or about interest rates.

Of course, we may want to introduce the idea of cooking by recipe or opening a bank account. However, we should not expect such activities to help with learning math; these activities should be—in fact must be—an application of math that is already learned.

Following are some ideas for using relevant materials in authentic ways.
Take a walk.

- Find a building made of bricks or concrete blocks and ask the learner 1) to make an estimate of how many bricks or blocks are in one wall, and in the whole building; and 2) to find a defined area on the building where there are about 100 blocks or bricks, and another defined area where there are about 1000 blocks or bricks. A “defined area,” is one that is easy to see, for example, “the whole side wall” or “the part of the wall to the left of the doorway” or “the blocks below the white line.” Make a sketch of these areas, and label them. This is a way of establishing a mental picture of the relative size of 100 and 1000.

- Ask learners to make a list of the math problems that someone had to solve to make the street look the way it does. (How many tulip bulbs do I need to fit into the flowerbed at 30 cm apart? How many steps do I need to cut so the staircase will reach the door? How much lumber do I need to buy for this fence? How many panels for that fence? How much paint do I need to cover the walls of this house? What angle do I cut for the pitch of the roof?) Just list the questions—no answers required.

- Make a list of questions that learners can answer about a given area, for example, a square block, or a three-block strip of a street. Ask about such things as the number of single or multiple family dwellings or the kind of businesses on the street; when the learners have answered the questions by walking the area, ask them to categorize the data and figure out proportions or percentages. For example, in a business district, to sort all the businesses into categories (food, retail, financial, medical, legal, government, etc.) and then to write proportions or percents to describe their data. Finally, they can make charts or graphs to display the data. If you ask different learners to go to different areas, they can compare their results.

Take a trip to the do-it-yourself store.

Take learners to the lumberyard or the home renovation store with a sheet of those questions you find in every text that teaches area and perimeter. Have them answer the questions using material found in the store. For example, take the question about how much flooring to buy for a particular size of room. They should pick out the flooring they want, figure out the area of the room in the problem, check that the units in their problem are the same as the units the flooring is sold by (square feet, square yards, square meters), and figure the cost of the flooring. Let them do the same thing for the problem about how many panels of fencing to buy. A digital camera is a welcome addition here, so that they can attach a picture of the flooring or the fence to their solution to the problem.

Call ahead to the store to let them know you’re coming, and you may get co-operation and help from the staff at hand.
Find out about numbers at your program.
If you are working with a group, divide them into teams to find the information about your program. If you are working with one learner, decide how much and what kind of information you will look for. It may be too big a job for one person to cover all the bases listed below.

- Some learners could go on a tour with the person in charge of facilities, for example, to find out about numbers in that job, such as: How much toilet paper does the school use in a year? How many photocopies get made in a month? How many cleaners work for how many hours a week/year?

- Other learners could talk to an administrator to find out about how many instructors work there, how many support staff, how many administrators. Who has worked there the longest? Who has worked there the shortest time? How many learners are there? What is the total budget the person administers?

- Other learners might go to the cafeteria to find out how many cups of coffee are served per day, or how many hamburgers; what days are busiest, slowest; how many times the dishwasher is run, how much detergent is used, etc.

When the data has been gathered, make charts or graphs to show the information, and display it where all can see, and/or put it in the newsletter.

Go to the supermarket.
Ask learners some questions about shopping for food: How do you shop? How do you decide what to buy or where or when to buy it? Do the stores charge more on the day that cheques are sent out? What are some questions you might like answered?

Then go together to find some answers on site. Some learners may be banned from some stores, so, before you set out, ask the learner(s) what would be a good place to visit. Phoning ahead may get you some helpful cooperation from store staff.

Investigate larger purchases.
Talk with learners about their options to buy furniture and other large ticket items. Where do they find a couch or table when they need one? Look at prices at thrift shops, second-hand and retail stores, and compare such options as lay away, rent-to-own, credit card and no-money-down-no-payment-until-sometime-in-the-future.

Buy a Lotto 649 ticket.
In an activity posted on the NSSAL Instructors’ Website, David Pilmer presents an opportunity for learners to explore the possibility of winning while playing Lotto 649. Learners are asked to pick a number, and to suppose they have played that number since the Lotto first became available. They are given the winning numbers of every draw in the history of the Lotto, and asked to figure out how much they won over the years, and how much they paid in. You will find this activity on page forty of a resource called
Bring real life to the classroom.
Sometimes issues of privacy and messiness make it difficult to ask learners to bring their real life problems to math class. However, life inside the classroom is also real, and everyone can see what the parameters are. Look around for problems in your program that learners could take part in solving. Where is your program spending money that learners might have some input? Some examples follow, but really, it is a question of keeping your ears and eyes open.

Decorating or renovating
This could be done with a group of learners, or with one. It would be too much of a fantasy to ask everyone to imagine that they were going to spend $1000 decorating a room in their house, or even $200. Yet they can see that some part of the building where classes are held needs work—perhaps the foyer is shabby, or the coffee room doesn’t have enough seating, or the smoke pit is damp and cold, or somewhere there is not enough storage, so boxes of various supplies are sitting around the hallways or classrooms.

Take a space in your program—the lobby or lounge area, the cafeteria or a classroom, or an outside area where learners hang out, and ask them to make a plan for refurbishing it. Ask them to make a scale diagram of the space and show placement of various elements they would bring in. Go on a field trip to price paint, hardware, furnishings, etc., with a camera so learners can take pictures of things they would like to incorporate into their design. Back in class, ask them to figure out how much things would cost and present a budget and their design to their classmates. The class could decide to present some or all of their ideas to administrators, or to other learner groups, although the administration is likely to be a more sympathetic audience—whether or not they actually agree to some of the changes.

Fundraisers
Learners might like to be involved in a fundraising activity for your program. This may involve lots of math, so be sure to allow time for learning as you are doing the activities.

- What will you raise the money for? Computers, classroom equipment, field trips, special programs? How much money do you need to reach your goal?
- What will your expenses be for the fundraiser? Make a budget in advance of what you will need to spend. Keep track of expenses, and compare your actual spending with your budget.
- Make an estimate of how much time it will take to carry out the activity. Keep track of the hours people put in as you go along.
• Evaluate the activity when it is over. Subtract the amount spent from the amount raised to figure out your net proceeds. Then figure out how much you made per hour of work done. List the non-monetary benefits of the activity. Was the fundraiser worth the time, energy and money you spent?

How much is a million? A special fundraising project

Collecting a million pennies would raise $10 000 for your program. Making a collection of a million pennies would involve publicity, since the wider community is needed to help make the collection, which means that learners can talk about the process of showing what a million is. They are involved in making a system for storing, sorting, and displaying the pennies they are collecting. It takes time, so the immensity of the number has time to set in, and it provides a repetition of adding and multiplying smaller amounts—10 tens make 100 and 10 hundreds make 1000 many, many times during the process of collecting a million of anything. Other questions: How far are we from our goal? How many do we collect on average per day, per week, per month? If learners put up a table at the mall to collect pennies, and stay there for three hours, how many do they collect per hour?

Adding, multiplying, keeping a running total, checking figures, subtracting to see how far there is to go to the goal—all these are here in abundance. There is lots of room for ratio and proportion in this project: how much do 100 pennies weigh? How much will 1 000 weigh? 10 000? 100 000? If it takes three minutes to count 100 pennies and put them in the plastic case, how long will it take to deal with 1 000 pennies? How long will it take to wrap the million pennies? Many more questions will come up.

There is also lots of interesting and meaningful work in reading and writing skills, too: talking to the press and to members of the community, making posters and flyers about the project, writing a report for the newsletter, and so on.
Using Manipulatives to Teach for Understanding

Making math more concrete is important for many adult learners who have difficulty following verbal explanations because they are not fluent with oral language, and may also have difficulty reading; it is also important for learners who are visual or kinesthetic learners. There are several ways of making math concepts more concrete:

- pictorial representations, such as diagrams and drawings
- manipulatives which are three dimensional, and may include real things (e.g., measuring tapes or mugs and cups or apples) or bits of plastic or wood specifically designed for teaching math (e.g., fraction circles, fraction strips or base ten blocks)
- online supports (applets), which are computer models of real objects, pictures or manipulatives, and which are often interactive

Why use manipulatives?
The obvious answer is that they show, in concrete terms, numbers and processes that are represented in abstract terms when we use numerals. More than that, however, manipulatives serve several purposes. First, manipulatives slow down the action, so learners have more time to understand and absorb ideas. Second, learners control the pace of the work when they control the manipulatives. This allows them to move into more abstract representation of the problem as their understanding grows, at their own speed. Third, manipulatives help learners remember what they did, because they are always visual and kinaesthetic, and often involve talking as well. Finally, learners usually get the answer right when using manipulatives, which may help them learn to correct the errors they make when doing calculations.
Number Sense

Developing number sense should be behind every numeracy activity we do with learners. It is not something to be taught separately, but rather it is part of every math idea and calculation. Number sense includes:

- A sense of the size of a number (How much are we talking about?)
  
  e.g. Order the following numbers from smallest to largest.
  
  (a) 1010, 1001, 1101, 1011, 1100
  
  \[ \frac{7}{8}, \frac{1}{7}, \frac{4}{9}, \frac{5}{6}, \frac{2}{3}, \frac{1}{15} \]
  
  (b) 0.09, \[ \frac{7}{5}, -1, 0.4, \frac{8}{9}, -0.3 \]
  
  (c)

- The ability to compute mentally (How can I figure it out in my head?)
  
  e.g. Calculate each of the following mentally.
  
  (a) 234 + 99
  
  (b) 3 x 48
  
  (c) Take 15% of 40.
  
  \[ \frac{1}{9} + \frac{5}{9} + \frac{1}{4} + \frac{3}{9} \]
  
  (d)

- The ability to make an estimate (What do we think the answer is close to?)
  
  e.g. Estimate each of the following.
  
  (a) 129+9+32
  
  (b) 20.05+15.9-0.09
  
  (c) 16% of 49
  
  \[ \frac{2}{12} + \frac{1}{30} \]
  
  (d)

- The ability to judge whether the answer is reasonable (Does my answer make sense?)
  
  e.g. There are 100 coworkers planning to charter buses for a concert in Moncton, New Brunswick. If each bus can take 40 people, Jamie concludes that they must charter 2.5 buses. Is this a reasonable answer? Why or why not?
As learners’ number sense develops, confidence in their ability to understand and do math increases; as number sense increases, the likelihood of a useful answer to real life mathematical situations increases, and learners’ sense of the usefulness and pervasiveness of math grows. (*Number Sense*, a publication by C. David Pilmer from NSSAL, deals with developing number sense and includes many activities which can be photocopied for learners.)
Metacognition (Learning about Learning)

Many adult learners are not used to being active learners in school subjects, and do not understand that they need to take an active part in the proceedings if they are going to succeed. Even those who are very actively involved in learning skills and information in other parts of their lives, may, when they get to school, sit back and cross their arms as if to say, “So teach me.” As well, many of them are unaware of the way math skills and concepts build on each other, or the kind of problem-solving thinking that is involved in doing math. They may think that it is some kind of magic rather than a set of skills and a way of thinking that makes some people “good at math” while others are not.

When they become aware that it is not just “magic,” they have some way of thinking about learning math. Part of the job of the numeracy practitioner is to make the learner aware of the separate skills, the names of the skills, and when and how to use them. Some of the teaching techniques that follow later, particularly the “think-aloud” on page 21 and marking for confidence on page 21, help learners articulate the skills they are learning.

Another important part of learning about learning is for learners to become aware of their learning strengths and preferred learning styles. There are questionnaires online and on paper to assess learning styles and multiple intelligences, as we saw in the module “Literacy, Adult Learners and YOU!” It is helpful to do more than one questionnaire, because usually the results will be a little different from each, and the learners will be able to reflect on which they find more accurate. When the learning strengths and styles of the learners are known, the practitioner can plan instruction based on that information, and each learner can choose study methods that will be most fruitful.

Evaluate the Strategies
A useful activity is to have the learner(s) evaluate the teaching techniques you are using. For example, you might show them two or three different ways to work on remembering the times tables. Work for 15 minutes with each technique, with breaks in between. Talk with them about which strategy works best. Repeat the process at your next session, using the same techniques, but different tables, and talk again about how the strategies are working. Let them decide which strategy or strategies seem most helpful, which they will continue to use, and which they will drop.

By inviting learners to evaluate the strategies, you ask them to reflect on their learning, and to be aware of what and how they are learning. When they evaluate the strategies, they are in a strong position of self-knowledge and control, rather than in the weaker position of being evaluated (and perhaps found wanting) by the strategy.
Automaticity

You may call it over-learning, or learning to mastery level, or having the facts at your fingertips, but whatever you call it, it is important to help learners get to the point of automaticity for any math skill we teach. No matter how well you know the concepts of addition, regrouping or carrying, you cannot do addition well if you do not automatically know that $9 + 3 = 12$. And if you go on to multiplication without being automatic in your addition facts, you make mistakes in doing multiplication with two or three digits; and if you go on to division the lack of ease with the number facts interferes with remembering the complex procedure of long division. Then you go on to fractions without addition/subtraction facts and without automatic times tables, and those things take up so much of your memory and your energy that you cannot think about common denominators and other aspects of fractions operations.

Making time and room for automaticity means that a lot of math time is taken up with things that the learner finds easy to do. It means spending hours playing the games that burn the “number facts” into the mind. For many of our learners who have been drilled with flashcards without effect, it means finding another way to internalize the facts. Two examples are given in later in this handbook, “Whole Numbers at the Board” (page 24) and “Fractions as the Board” (page 32).

The danger here is the temptation to move on too soon.

A locally developed resource that assists learners in developing automaticity as it pertains to the recall of addition and multiplication facts can be found at the NSSAL instructor’s website. It is titled Mental Math for Level 1. The author, David Pilmer, makes four recommendations about mental math:

- Mental math activities should be gradual and continuous: therefore we suggest that these types of activities could serve as a warm-up activity at the beginning of each class.
- Strategies and the associated facts should be presented after learners have obtained conceptual understanding of the operations with whole numbers. This may require that the learners examine patterns, use manipulatives, and/or work in familiar contexts before the mental math strategies are introduced.
- …Merely saying the question aloud may not serve the needs of all learners. A visual component must also be provided either on the board, overhead, or flash cards.
- …We suggest using small handheld white boards. Learners can write the solution to the problem on the board and then turn the board in the instructor’s direction. This technique allows the instructor to quickly scan the responses and ascertain the level of student understanding. This method eliminates the possibility of embarrassing an adult learner whose response is incorrect, but still allows for discussions of correct solutions. (Pilmer, 2007, page 4)
Some learners don’t get to the point of being automatic in their grasp of facts or processes, and after some time, practitioner and learner need to make some hard decisions. Will you abandon the work of learning these things, and to learn to use and rely on a calculator? Both learner and practitioner need to be aware of the consequences of this—for example, will it mean that the learner will never be able to access further training? If the decision is made to go over to the calculator for all operations, then the need for skills of estimating and assessing the reasonableness of the answer comes into full play.
Assessment

When you teach math to adult learners, you will be constantly assessing the learners’ skills and knowledge. This kind of assessment helps you make good decisions about teaching: What skills or knowledge does the learner have to cope with the next section of the work? What is missing? What kind of errors are being made? The numeracy practitioner needs to know the answers to these questions in order to plan for teaching and review.

Assessment can be non-threatening when handled correctly. For example, open-ended questions are questions that have a variety of acceptable answers ranging from simple to complex. This type of question can be accessible to a wide range of learners yet still inform the instructor or tutor about the learner’s level of understanding of the concept. Consider the following open-ended question and the possible range of responses.

Find two fractions that add to give you one.

**Possible responses:**

\[
\frac{1}{2} + \frac{1}{2} = 1 \quad \frac{3}{6} + \frac{3}{6} = 1 \quad \frac{4}{8} + \frac{5}{10} = 1 \quad \frac{5}{7} + \frac{2}{7} = 1 \quad \frac{3}{9} + \frac{4}{6} = 1
\]

(Simple) (Complex)

Another non-threatening assessment technique relies on the use of mini white boards. Suppose you wanted your learners to complete a series of mental math questions and share their correct strategies with other learners. Learners who are confident with their math abilities are likely to volunteer their answers and explanations. Learners who lack such confidence are unlikely to participate in the discussion. To alleviate this problem, the instructor can ask learners to respond to the question on their mini white board and then hold the board up so that only the instructor can see the answer. The instructor can then scan all the responses to see how individual learners are making out with the question, and only ask for responses from learners who have the correct answer. Those with an incorrect answer are not forced to respond and therefore not placed in a potentially embarrassing situation.

Also, the learner needs to be involved in assessment. When you use relevant materials in real life situations, it is easy for learners to make an assessment of their learning. When you are working from a text book or a worksheet, it is more difficult for them to make their own assessment of the learning, because they don’t have a well-developed sense of how it’s “supposed to be done.” “Marking for Confidence” (page 21) helps learners articulate the skill they are learning, while at the same time focusing on what they are doing right.

When you are thinking about sharing your assessment with a learner, ask yourself: Is the feedback clear and to the point? Can the learner use the feedback to make progress? Will the feedback motivate the learner to continue to work and improve? If the answer to any of these questions is no, think again about giving the feedback.
Teaching Techniques

Learning Journals
Learning journals or learning logs are often used to provide a way for learners to reflect on their learning—what they have learned, what they are working on now, what they know about themselves as learners, what problems they are encountering, their ideas for finding solutions to these problems, and so on. Sometimes there is a disconnect between writing in a learning journal and passing the course. Learners notice that writing in the journal takes time and effort, but the tests on each section or on the whole course do not reflect that work. One way to remedy that disconnect is to put an essay question on each test, which asks learners to think and write about something mathematical in a manner similar to a journal entry.

Sometimes space for journal writing is provided in a math book the learner is using, or they may use a small notebook or a special section of their binder. Provide time at each session for them to write in their journal. You can ask a question as a topic for the journal writing that day, or leave it to the learner to write about the lesson. Some sample questions: What did you work on today? How is it going? What did you learn about ________? What made the lesson clear for you today? What questions do you still have? Did anything surprise you today?

You should read the journals daily or weekly, so you are aware of the learner’s progress; you may make it a response journal by adding your own comments or questions. Be sure to find out in advance about how the learner would like you to respond—on the page itself, or on a sticky you can attach to the page.

One advantage of a learning journal is that the learner can look back over the pages and see progress, patterns of problems and evidence of strategies that work over time. However, many learners find writing itself such a chore that the value of the math journal is discounted by the difficulty of expressing the thoughts in writing. If you are working one-on-one with such a learner, you could write the journal as the learner dictates it to you, as a language experience activity. Or you could make the reflection process oral, which works especially well with a group. Ask everyone to sit in a circle so they can see each other, and go around so that each person gets a chance to say what they learned that day. A good question is “What did you learn today and where will you use that learning? You should answer too—it may be your learning about math, your learning about teaching, or your learning about the learners you work with.

As people are finishing up and leaving individually, I stand near the door to say good-bye to them. I ask them what they learned today, and how things went. The question reminds them that I expect them to learn something each day, and that I expect them to notice what they are learning. I think it promotes a more active attitude to learning, and it shows my interest in them. And I learn things about my teaching I never would have realized if I hadn’t asked!

Kate
The Think-Aloud
There is an example of a math think-aloud in the teaching toolbox:
http://alt.ns.literacy.ca/file.php/29/kate_thinkmath_video.htm

This technique is just what it says it is—thinking out loud while doing a math problem, so the learner gets an insight into how you think. Many learners have misconceptions about math thinking: they may think that people who “know” math can do any problem just by looking at it; they may be unaware of the separate steps and skills used to solve a problem. Thinking out loud helps dispel these misconceptions.

To prepare to do a think-aloud, go over all the separate steps and skills in your mind, so that when you are with the learner, you will be able to demonstrate how you think about solving a problem. For example, look at a word problem, read it over once, then read it again more slowly. Is there information somewhere else on the page that you will need to do the problem? How do you decide whether to use a drawing or a chart to sort out the information in the problem? Talk about looking at the units as well as the numbers, so you know if you have to convert some of the information. Ask questions about the information in the problem, and talk about the implications of the information. (e.g., “every day for a week…that means seven times.”) How do you figure out what the problem is asking? Make a plan for solving the problem, and talk about why you think that plan will work. Make an estimate of the answer. Carry out the plan, then compare with your estimate. Decide if your answer is reasonable.

After you have prepared yourself by thinking about all the details, offer to show the learner(s) how you would tackle a question. At first, the learner(s) will likely concentrate on the problem itself, but after you have done a couple of think-alouds, ask them to listen to you think aloud again, and concentrate on the steps and skills you use, rather than on the content of the particular problem. When you have finished the think-aloud, make a list of the steps and skills you used and discuss them.

Marking for Confidence
There is an example of marking for confidence in math in the teaching toolbox:
http://alt.ns.literacy.ca/file.php/29/Kate-math-confidence-video.htm

The following material is taken from a podcast by Kate Nonesuch:
http://alt.ns.literacy.ca/file.php/46/MarkingForConfidence.pdf

Marking for confidence is a way of giving feedback to students that increases their confidence in their ability to tackle similar problems or situations in the future. When I mark for confidence, I focus on the parts of the assignment or process that the learner is doing well. I comment on those parts, and help the learner articulate exactly what they are doing at that point, and why their procedure is correct.

Marking for confidence is not the same as just being encouraging. If I say to a learner, “I’m sure you’ll be able to do these questions,” the student only
knows that I have confidence in her. She may or may not agree with me. She may not have confidence in herself. If I say to a learner, “great work,” he may believe that he did great work, but he doesn’t necessarily know what he did that made it great work, or how to do it again. Marking for confidence gives him a chance to see, articulate and remember the details of the work that make it “great work.”

I call it marking for confidence because it looks like marking, and because marking is something the learner is familiar with. But in reality, I turn the marking process into an opportunity for teaching.

People say, “But don’t you think that it’s good for people to see what they did wrong? They learn from their mistakes. We’re not doing them any good by being positive all the time!”

I say that learners are sometimes in a frame of mind to learn from their mistakes, and sometimes not. So if a learner asks me how to do something, or asks if something is right or wrong, I tell him. But if a learner is just handing something in, or says, “I’m finished. Can you mark this?” I’m not sure what her frame of mind is. Maybe she thinks it’s all right! If so, I don’t want to destroy her confidence. Maybe she knows it’s all wrong and she just wants to get it over with. Again, no indication that she’s in a frame of mind to learn from her mistakes.

If I see that the learner has made many errors, I don’t mark them at all. Instead, I take my share of the responsibility for asking the student to do something he was not prepared for, so he knows that his mistakes were part of a complex process that involve my explanations, his ability to attend to them, time pressures, his previous knowledge, my knowledge of his math level, and emotional factors. I might say, “I made a mistake with this—I’m sorry I asked you to do this sheet right away. I didn’t know you weren’t clear on how to do it.” Then he gets a chance to be generous with me. Maybe he’ll say, “That’s all right, Kate. Everybody makes mistakes sometimes.” Talk about role reversal! That is a great position for him to be in.

Maybe he’ll say, “I guess I should have asked more questions when you were explaining it before.” Again, that is a step forward as he recognizes his responsibility to get clear explanations, and articulates one strategy for doing it. So since we have established that that set of questions was not appropriate for him, we can start the teaching process again, and later I’ll produce a clean copy of the assignment for him to do, or an alternate set of questions.

If I marked every question wrong as a first step, I would not get either of those positive responses from the learner. Instead, I’d be left to deal with someone who was discouraged and possibly angry at himself or at me. Discouragement and anger leave little room for learning.

It is confidence that allows learners to make decisions, to decide how to tackle a problem, to believe they can tackle a problem. If they have no confidence, they will take no risks. If they believe they have failed at similar tasks in the past, they are unlikely to believe they can be successful with a
new assignment. Marking for confidence builds confidence, not a false sense of “I can do anything!” but a sense that the learner has a base of knowledge and ability that will serve him as he tackles new work.

**Process**

Ideally, you should respond as the learner works. When learners start to work on a page in their books or on a worksheet, don’t wait for them to finish and hand it in, rather move around immediately, marking each one as you go, then coming back when they have a few more questions done. If you are working with a group, when you find several people having the same difficulty, call the class together to clarify the work, and go over it again.

If you are working one-to-one to mark a learner’s work, your job is to be encouraging by pointing out what they have done correctly, and what evidence you see of good thinking and of learning. You also want to give the learner a chance to articulate what they are doing, to help them remember, and to give them control over the process.

So look at a page with the learner, find the first question that is correct, and mark it right. Take a quick glance at the paper to see if they got only a few, or many wrong. Ignore the ones that are wrong for the moment, and comment that the learner got some or many correct.

**If there are only a few wrong,** start with the first one that is correct. Ask the learner to explain how they did it, so they have a chance to “rehearse” the procedure and articulate it clearly. Repeat with the next correct one, and the next, until they can easily articulate the procedure. Then move to the first one that was wrong, and again ask the learner to explain what they did. Usually, they will find their mistake and correct it on their own. Mark it right, and acknowledge that they were able to find their own error. Continue with the next one wrong. At some point, you can mark all the correct ones with a checkmark, and ask them to independently correct all the ones that don’t have a checkmark beside them.

**If more than a few questions are wrong,** find the first one that is right, mark it right, and ask the learner if you can guess what they did. Go over the question, teaching and dialoguing with the learner about the method. Try to figure out from their talk where they went wrong in the other questions. Then present a new question to the learner and ask them to do it while you watch/coach. Then another. When you are sure the learner has the process in mind, offer a clean copy of the worksheet to do, or a new worksheet with similar problems.

**Working at the Board**

The following two techniques, “Whole Numbers at the Board” and “Fractions at the Board” are taken from *Changing the Way We Teach Math* (Nonesuch, 2006) and used by permission of the author. They are an informal way of doing a constant review and practice to promote automaticity; they also reinforce group bonding and cooperation. Even if you are working with only one person, working at the board is preferable to doing it on paper, because it is easier for the learner to erase on the board, and writing on the
board uses large arm muscles and allows students to walk around more easily, to be freer in their bodies, which is good for kinesthetic learners.

Whole Numbers at the Board

(Time: 15-20 minutes a day)

The following series of whole number activities allows you to check how fluent your learners are with reading and writing numbers to one million, and gives them lots of practice in reading numbers and in working with place value. It also shows the value of review and over-learning; learners will notice as it gets easy to answer questions similar to the ones that were difficult the week before. Learners should find most of these questions easy, because they are doing many examples of similar questions until it seems easy. If every question is hard, they will be reluctant to go to the board. Every day only a few questions should present some challenge. You can tailor the questions to suit your learners, backing up a step when they are having difficulty, and asking them to explain their thinking on questions they are finding easy to do.

When learners can give a correct response quickly and easily, that is the time to ask them to explain their thinking, since usually the skill of talking about math lags behind the skill of doing math. So while some are finding a particular exercise challenging to do, others, who find it easy to do, can practice the next challenge—explaining their thinking.

When learners are familiar with the tasks, ask for a volunteer to “be the teacher” and read the instructions for other students to follow. The challenge of checking that learner responses are correct, while running the group process of reading answers and noticing patterns, will be an interesting math experience for those whose skills in doing the math are more advanced than most of the group.

Process

Each day, ask learners to go to the board to read and write some numbers which you will call out. Encourage them to look around at other learners’ work, stand beside someone who is good at math, get a long piece of chalk, and so on. They should do whatever they need to be comfortable at the board. Especially, make sure there is an eraser between every pair of learners. Having to ask for an eraser would call attention to their errors. We want them to take a risk, so make sure they can quickly and quietly erase answers that are wrong.

Every day, the work follows the same pattern:

1. As you call out the first instruction, learners will write the number at the board. For example, “Write a three-digit number with 5 in the ones place.” There will be a variety of correct answers.
2. Quickly look around and make sure that every response is correct, helping individuals as necessary.
3. Ask one of the learners to read the number written by the learner on their right; make sure it is loud enough for everyone to hear.
4. Ask the next learner to read the number written by the learner on their right, and so on until every number has been read.

5. Notice, comment on or ask about any patterns you see. For example, which number is largest? Smallest? What is the largest possible correct answer? Smallest possible? When many learners write the same number, is it because they don’t have many choices of correct answers? (For example, write the largest possible three-digit number.)

6. Go on to the next instruction, and the next.

7. Finally, thank learners for their participation and their excellent work, and invite them to sit down.

Day 1

- Rule out the use of 0 as a first digit in today’s work.
- Write a one-digit number that is bigger than 5. Ask each learner to read the number written by the learner on the right. Ask: Who wrote the smallest number? Who wrote the largest number? And so on as suggested above.
- Write a one-digit number smaller than 7. Ask each learner to read the number written by the learner on the right. Questions as above.
- Write a two-digit number with 7 in the tens place. Ask each learner to read the number written by the learner on the right. Questions as above.
- Write a two-digit number that comes between 17 and 25. Ask each learner to read the number written by the learner on the right. Questions as above.
- Write a two-digit number with a 0 in it. Ask each learner to read the number written by the learner on the right.
- Write a three-digit number with 9 in the hundreds place. Ask each learner to read the number written by the learner on the right.
- Write a three-digit number that comes between 127 and 130. Ask each learner to read the number written by the learner on the right.
- Write a three-digit number with a 0 in it. Ask each learner to read the number written by the learner on the right.
- Write a three-digit number with two zeros in it. Ask each learner to read the number written by the learner on the right.

Day 2

(Preparation: Review or teach reading numbers larger than 1 000.)

- Rule out the use of 0 as a first digit in today’s work.
• Write a three-digit number with 5 in the tens place. Ask each learner to read the number written by the learner on the right. Ask questions of the whole group, for example: Which number is largest? Smallest? What is the largest possible correct answer? Smallest? When many learners write the same number, is it because they don’t have many choices of correct answers?

• Write a three-digit number with 5 in the hundreds place. Ask each learner to read the number written by the learner on the right. Ask questions, as before.

• Write a three-digit number with 5 in the ones place. Ask each learner to read the number written by the learner on the right. Ask questions, as before.

• Write a four-digit number. Ask each learner to read the number written by the learner on the right. Ask questions, as before.

• Write a four-digit number with 7 in the tens place. Ask each learner to read the number written by the learner on the right. Ask questions, as before.

• Write a four-digit number that comes between 1 700 and 1 800. Ask each learner to read the number written by the learner on the right. Ask questions, as before.

• Write a four-digit number with three zeros in it. Ask each learner to read the number written by the learner on the right. Ask questions, as before.

• Write a four-digit number with two zeroes in it. Ask each learner to read the number written by the learner on the right. Ask questions, as before.

• Write a four-digit number with a 0 in it. Ask each learner to read the number written by the learner on the right.

• I will give you two digits, 8 and 3. Write all the numbers you can, using those two digits. Write the numbers in order from smallest to biggest.

Day 3

• As usual, rule out the use of 0 as a first digit in today’s work.

• Write a four-digit number with 7 in the thousands place. Ask each learner to read the number written by the learner on the right. Ask questions of the whole group, for example: Which number is largest? Smallest? What is the largest possible correct answer? Smallest? When many learners write the same number, is it because they don’t have many choices of correct answers?

• Write a five-digit number. Ask each learner to read the number written by the learner on the right. Ask questions, as usual.

• Write a five-digit number with 7 in the ones place. Ask each learner to read the number written by the learner on the right.

• Write a five-digit number that comes between 17 000 and 18 000. Ask each learner to read the number written by the learner on the right.

• Write a six-digit number with a 0 in it. Ask each learner to read the number written by the learner on the right.
• Write a six-digit number bigger than 500 000. Ask each learner to read the number written by the learner on the right.

• Write a six-digit number with 4 in the thousands place. Ask each learner to read the number written by the learner on the right.

• I will give you two digits, 6 and 4. Write all the numbers you can, using those two digits. Write the numbers in order from smallest to biggest.

• I will give you three digits, 5, 6 and 9. Write all the numbers you can, using those three digits. Write the numbers in order from smallest to biggest.

• Doubling: Start with 5. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)

• Repeat, starting with 2.

Day 4

• Again, and on all the following days, rule out the use of 0 as a first digit in today’s work.

• Your questions for the first set could focus on explaining the thinking that led to the answer. How did you decide on your answer? What would the next number bigger (or smaller) be? Does that meet the criteria? Looking at all the answers, what pattern in 0 and 9 do you see?

• Write the biggest one-digit number possible. Ask someone to read it.

• Write the smallest two-digit number possible. Ask someone to read it.

• Write the biggest two-digit number possible. Ask someone to read it.

• Write the smallest three-digit number possible. Ask someone to read it.

• Write the biggest three-digit number possible. Ask someone to read it.

• Write the smallest four-digit number possible. Ask someone to read it.

• Write the biggest four-digit number possible. Ask someone to read it.

• Write the smallest five-digit number possible. Ask someone to read it.

• Write the biggest five-digit number possible. Ask someone to read it.

• Write a six-digit number with 7 in the thousands place. Ask each learner to read the number written by the learner on the right.

• Write a three-digit number bigger than 995. Ask each learner to read the number written by the learner on the right.

• Write a seven-digit number with 6 zeros in it. Ask each learner to read the number written by the learner on the right.

• Write a seven-digit number. Ask each learner to read the number written by the learner on the right.
• Write a seven-digit number larger than the one you just wrote, if possible. Ask each learner to read the number written by the learner on the right.

• I will say a number. Write the next number bigger than the one I say.

9 19 29 39 59 79 99 199 299 399 499 599 699 799 899 999

• I will give you two digits, 1 and 9. Write all the numbers you can, in order from smallest to largest.

• I will give you three digits, 7, 2 and 5. Write all the numbers you can, using those three digits. Write the numbers in order from smallest to biggest.

• Doubling: Start with 5. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)

• Repeat, starting with 2.

• Repeat, starting with 3.

Day 5
• Write the smallest two-digit number possible. Read it to the person next to you.

• Write the smallest three-digit number possible. Read it to the person next to you.

• Write the smallest five-digit number possible. Read it to the person next to you.

• Write the smallest seven-digit number possible. Read it to the person next to you.

• Write the smallest eight-digit number possible. Read it to the person next to you.

• Write a three-digit number with 3 in the tens place. Read around.

• Write a four-digit number bigger than 4 000. Read around.

• Write a six-digit number. Read around.

• Write a seven-digit number. Read around.

• I will say a number. Write the next number smaller than the one I say.

8 29 31 40 50 80 100 200 400 500

• I will give you three digits, 1, 2 and 3. Write all the numbers you can, using those three digits. Write the numbers in order from smallest to biggest.

• I will give you four digits, 3, 7, 1 and 5. Write all the numbers you can, using those digits. Write the numbers in order from smallest to biggest. (This is much harder than the previous question; you might want to give some learners four digits and others only three.)

• Doubling: Start with 5. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)

• Repeat, starting with 2.

• Repeat, starting with 3.
Day 6
- Write the smallest seven-digit number possible. Read it to the person next to you.
- Write a three-digit number with 9 in the hundreds place. Read around.
- Write a four-digit number between 8 000 and 9 000. Read around.
- Write a three-digit number between 125 and 128. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 7. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
  - Repeat, starting with 2.
  - Repeat, starting with 3.
  - Repeat, starting with 11.

Day 7
- Write the smallest seven-digit number possible. Ask someone to read it.
- Write a three-digit number with 9 in the hundreds place. Read around.
- Write a five-digit number between 29 000 and 30 000. Read around.
- Write a six-digit number between 200 000 and 300 000. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 7. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
  - Repeat, starting with 20.
  - Repeat, starting with 30.
  - Repeat, starting with 11.

Day 8
- Write the largest seven-digit number possible. Read around.
- Write a five-digit number with 9 in the hundreds place. Read around.
- Write a five-digit number between 54 000 and 55 000. Read around.
- Write any number with 1 in the thousands place. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 7. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
  - Repeat, starting with 2.
  - Repeat, starting with 3.
• Repeat, starting with 11.

Day 9
• Write the largest three-digit number possible.
• Write any number with 9 in the thousands place. Read around.
• Write a number between 4 857 and 4 859. Read around.
• Write a hard number to read. Read around.
• Doubling: Start with 5. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
• Repeat, starting with 11.
• Repeat, starting with 13.

Day 10
• Write the largest six-digit number possible. Read it to the person next to you.
• I’ll give a number; you write the next whole number bigger.
  17   39   109   999   1 045   10 000
• Write a number between 587 and 600. Read around.
• Write a hard number to read. Read around.
• Doubling: Start with 2. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
• Repeat, starting with 5.
• Repeat, starting with 3.

Day 11
• Write the largest seven-digit number possible. Read it to the person next to you.
• I’ll give a number; you write the next whole number bigger.
  33   59   709   9 999   1 045   100 000
• Write a number between 1 110 and 1 115. Read around.
• Write a hard number to read. Read around.
• Doubling: Start with 3. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
• Repeat, starting with 7.
• Repeat, starting with 9.

Day 12
• Write the largest seven-digit number possible. Read it to the person next to you.
• I’ll give a number; you write the next whole number bigger.
  38  92  799  1000  9999  100000
• Write a number between 1256 and 1260. Read around.
• Write a hard number to read. Read around.
• Doubling: Start with 2. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
• Repeat, starting with 11. Repeat, starting with 13.
• Day 13
• Write the largest four-digit number possible. Read it to the person next to you.
• I’ll give a number; you write the next whole number bigger.
  56  99  709  9999  200000  1000000
• Write a number between 12600 and 12700. Read around.
• Write a hard number to read. Read around.
• Doubling: Start with 10. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
• Repeat, starting with 100.
• Repeat, starting with 9.
• I will give you a number. Cut it in half, then cut the answer in half and keep on going until you come to a number that you can’t cut in half evenly. Start with 1000.
• Repeat, starting with 1200.
• Repeat, starting with 800.

Day 14
• Write the largest three-digit number possible. Read it to the person next to you.
• I’ll give a number; you write the next whole number bigger.
  909  127  6425  9999  7777  99000
• Write a number between 11700 and 11800. Read around.
• Write a hard number to read. Read around.
• Doubling: Start with 200. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
• Repeat, starting with 50.
• Repeat, starting with 30.

Note: The material in the first 14 days corresponds perfectly with Level I, but the subsequent days are more like Level II, due to the work with decimals.
Day 15
(Preparation: Teach or review short method of multiplying by 10.)

- Multiply by 10: 75 155 250 2 500 17.9 26.7 27.89 156.9876
- Divide by short division. Start with 240; divide by 2 and keep dividing the answer until it can’t go evenly any more.
- Repeat, starting with 1 000.
- Repeat, starting with 1 400.

Day 16

- Multiply by 10: 35 127 360 7,350 27.2 39.6 49.89 257.3694
- Divide by short division. Start with 650; divide by 2 and keep dividing the answer until it can’t go evenly any more.
- Repeat, starting with 96.
- Repeat, starting with 84.

Day 17
(Preparation: Teach or review short method of dividing by 10.)

- Multiply by 10: 46 777 365 5 600 26.9 179.2 139.89 1256.3742
- Divide by 10: 30 120 50 100 900

Day 18

- Multiply by 10: 29 760 127.3 130.35 127.96 3.7524
- Divide by 10: 10 25 100 120 30.1 25.20

Following Days
Many days of board work follow this pattern—multiplying and dividing by 100; short and long division, finding factors, and so on. Working at the board is a way to review, practice and consolidate work presented in tests and lessons.

Fractions at the Board
(Time: 15-20 minutes a day)
The following series of activities with fractions gives learners practice with fractions, allows you to check their understanding and their ability to manipulate fractions, and gives them a chance to articulate what they know. It also shows the value of review and over-learning; learners will notice as it gets easy to answer questions similar to the ones that were difficult the week before. They should find most of these questions easy, because they are doing many examples of similar questions until it seems easy. If every question is hard, they will be reluctant to go to the board. Every day only a few questions should present some challenge. You can tailor the questions to suit your learners, asking more questions similar to ones they are learning, or skipping some questions that they are
finding too easy or too difficult at the moment. For some learners you can repeat one
day’s activities for several days, using different numbers.

When learners can give a correct response quickly and easily, that is the time to ask them
to explain their thinking, since usually the skill of talking about math lags behind the skill
of doing math. This means that learners of different abilities can be working on the same
content area with the same questions: for some at a lower level, just doing the work is the
job at hand; for others at a higher skill level, articulating the process is the job that
requires work and keeps the interest high.

When learners are familiar with the process, ask a learner to “be the teacher” and read the
instructions for the others to follow. The challenge of checking that learner responses are
correct, while running the group process and noticing patterns, will be interesting math
experience for some learners whose skills in doing the math are more advanced than most
of the group.

**Process**

Each day, ask learners to go the board to read and write some fractions. Encourage them
to make themselves at home, to look around at other learners’ work, to stand beside
someone who is good at math, to get a long piece of chalk, and so on. They should do
whatever they need to be comfortable at the board. Especially, make sure there is an
eraser between every pair of learners. Having to ask for an eraser calls attention to their
errors. We want them to take risks, so make sure they can quickly and quietly erase their
answers if they are wrong.

**Every day, the work follows the same pattern:**

1. As you call out the first instruction, learners will write the fraction(s).
2. Quickly look around and make sure that every response is correct, helping
   individuals as necessary.
3. Ask learners to look around to see if their answer is the same as everyone else’s.
   Different answers may be correct, and provide an opportunity to talk about the
   process.
4. Repeat for each of the instructions on the day.
5. Thank learners for their participation and their excellent work, and invite them to sit
down.

**Day 1**

- Think about all the people in the room. What fraction are men? Write that fraction.
  Write the fraction of people who are women. What fraction are teachers? What
  fraction are learners?
- Think about the people at the front board. What fraction are men? Write that
  fraction. Write the fraction of people at the front board who are women.
- Think about the people at the side board. What fraction are men? Write that
  fraction. Write the fraction of people at the side board who are women.
• Suppose there were twelve people in the room and half were women. Write a fraction with 12 on the bottom that shows the fraction of women in that room. Correct. 6/12 is equivalent to 1/2.

• Suppose there were 100 people in the room and half were men. Write a fraction with 100 on the bottom that shows the fraction of men in that room. Correct. 50/100 is equivalent to 1/2.

• Write five fractions equivalent to 1/2.

• I’ll give you three fractions, 1/2, 1/10 and 1/4. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

Day 2

• Think about all the people in the room. What fraction are wearing glasses? Write that fraction. Write the fraction of people who are not wearing glasses.

• Think about the people at the front board. What fraction are wearing glasses? Write that fraction.

• Think about the people at the side board. What fraction are wearing glasses? Write that fraction.

• Repeat until everyone can write a fraction easily, asking for the fraction of people wearing shorts, wearing skirts, wearing black, or wearing watches, etc. Ask the question of the whole room, and then of one or more smaller groups of learners (for example, front board or side board), so that the denominators are not the same all the time. Sometimes include yourself, sometimes not, for example, “What fraction of the people in the room are wearing glasses? What fraction of the learners are wearing glasses?”

• Suppose there were six pieces of pizza left after the party, and half had mushrooms on them. Write a fraction with 6 on the bottom that shows the fraction of pieces that had mushrooms. Correct. 3/6 is equivalent to 1/2.

• Write five other fractions equivalent to 1/2.

• I’ll give you three fractions, 1/6, 1/2 and 1/100. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

• I’ll give you three fractions, 5/8, 1/8 and 3/8. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

Day 3

• Think about the people in this room. What fraction of the women are wearing black shoes today?

• What fraction of the men are wearing black shoes today?

• What fraction of the learners are wearing black shoes today?
• Repeat until everyone can write a fraction easily, asking for the fraction wearing shorts, wearing skirts, wearing black, or wearing watches, etc. Ask the question of the whole room, and then of one or more smaller groups of learners, so that the denominators are not the same all the time.

• Write five fractions equivalent to 1/2.

• I’ll give you five fractions, 1/6, 1/2, 1/4, 1/12 and 1/100. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

• I’ll give you four fractions, 4/7, 3/7, 1/7 and 7/7. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

• (Use this question after you have taught a process for generating equivalent fractions.) Write a fraction equivalent to 1/2 with a bottom number of 4. (Write the question up in the usual way so that people can see it. 1/2 = /4). Continue with a few more: 1/4 = /8; 3/4 = /8; 1/3 = /6; 2/3 = /6.

Day 4

• Ask them to write several fractions describing people in the room, e.g., What fraction of the men have their hair tied back? What fraction of learners have their hair tied back? What fraction of women?

• Write five fractions equivalent to 1/2.

• I’ll give you five fractions, 1/10, 1/2, 1/4, 1/5 and 1/20. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

• I’ll give you four fractions, 4/10, 3/10, 5/10 and 9/10. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

• Write some equivalent fractions: a fraction equivalent to 1/2 with a bottom number of 4. (Write the question up in the usual way so that people can see it. 1/2 = /4). Continue with a few more: 1/4 = /8; 3/4 = /8; 1/3 = /9; 2/3 = /12.

• I eat 3/4 of a chocolate bar. What fraction is left?

• The kids eat 5/8 of the pizza. What fraction is left?

• The team plays the first quarter of the basketball game. How much of the game is left?

• I do 1/3 of my homework. What fraction is left?

Day 5

• Draw three columns at the board. At the top of one write “less than 1/2,” at the top of the next, write “= 1/2,” and at the top of the last column write “more than 1/2.” I’ll give you some fractions. You write them in the correct column: 1/4, 7/8; 5/10, 3/4, 5/6, 1/12, 3/100.

• Write five fractions equivalent to 1.

• I’ll give you five fractions, 1/3, 1/10, 1/5, 1/9, and 1/2. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
• I’ll give you four fractions, 89/100, 39/100, 51/100 and 9/100. Write them in order from smallest to largest. If you like, draw some diagrams.

• Write some equivalent fractions, 1/4 = /12; 1/3 = /12; 2/3 = /12; 3/4 = /12, 5/6 = /12.

• I watch 3/4 hours of a one-hour program. What fraction is left?

• I eat 1/2 of an apple. What fraction is left?

• The kids use up 4/5 of the toothpaste. What fraction is left?

• I watch the first half of the soccer game. How much of the game is left?

• I have to read 10 pages for homework. I read 9 pages. What fraction of my homework is left?

• Reduce these fractions to lowest terms: 2/4, 4/8, 6/12, 8/16, 12/24.

Day 6

• Draw three columns at the board. At the top of one write “less than 1/2;” at the top of the next, write “= 1/2,” and at the top of the last column write “more than 1/2.” I’ll give you some fractions. You write them in the correct column: 5/10, 4/8, 1/100, 6/12, 9/14, 10/20, 3/6.

• Write five fractions equivalent to 1.

• I’ll give you five fractions, 1/5, 1/14, 1/7, 1/4, and 1/2. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

• I’ll give you four fractions, 5/8, 1/8, 3/8, and 8/8. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.


• I watch 1/4 hour of a one-hour program. What fraction of the program is left?

• We eat 2/3 of a pizza. What fraction of the pizza is left?

• I drive 9/10 of the way to Nanaimo. What fraction of the trip is left?

• I have five hours of homework to do. I work for three hours. What fraction of the work is left?

• The Yankees played 5 innings against the Red Sox. What fraction of the game is left?

• Reduce these fractions to lowest terms: 4/10, 2/12, 4/14, 6/9, 12/16, 7/21.

Note: All of the material for the first six days corresponds nicely with Level II. The subsequent days start looking more like Level III due to the operations with fractions.
Day 7
- Draw three columns at the board. At the top of one write “less than 1/2,” at the top of the next, write “= 1/2,” and at the top of the last column write “more than 1/2.” I’ll give you some fractions. You write them in the correct column: 4/5, 1/8, 91/100, 8/15, 7/14, 5/10, 6/6.
- Write five fractions equivalent to 1.
- Write five fractions larger than 1.
- I’ll give you three fractions, 1/6, 9/10 and 2/4. Write them in order from smallest to largest. Think about whether they are bigger or smaller than 1/2, or equivalent to 1/2.
- I’ll give you five fractions, 5/9, 1/9, 7/9, 2/9 and 9/9. Write them in order from smallest to largest.
- I drink 1/2 of my cup of coffee. What fraction is left?
- We walk 3/4 of a block. What fraction is left?
- There were 10 questions on the test. I got 7 right. What fraction did I get wrong?
- The teams played one period of hockey. What fraction of the game is left?
- Subtract: 1 – 1/2; 1 – 3/4; 1 – 2/3.

Day 8
- Draw three columns at the board. At the top of one write “less than 1/2,” at the top of the next, write “= 1/2,” and at the top of the last column write “more than 1/2.” I’ll give you some fractions. You write them in the correct column: 5/8, 1/21, 91/100, 7/13, 6/12, 9/18, 3/3.
- Write five fractions equivalent to 1.
- Write five fractions larger than 1.
- Write five fractions smaller than 1.
- Draw three columns. At the top of one write “less than 1,” at the top of the next, write “= 1,” and at the top of the last column write “more than 1.” I’ll give you some fractions. You write them in the correct column: 5/8, 4/4, 9/10, 3/2, 7/7, 4/1, 6/6.
- I’ll give you three fractions, 1/6, 10/10 and 6/4. Write them in order from smallest to largest. Think about whether they are bigger or smaller than 1, or equivalent to 1.
• I’ll give you five fractions, 5/3, 1/3, 2/3, 3/3 and 10/3. Write them in order from smallest to largest.


• The third quarter of the football game is over. What fraction of the game is left to play?

• Four team mates run a relay race. What fraction of the race does each person run?

• Reduce these fractions to lowest terms: 4/10, 7/21, 8/16, 6/18, 5/25, 7/56.

• Add: 1/2 + 1/2; 1/2 + 1/4; 1/2 + 1/8.

• Subtract: 1/2 – 1/2; 1/2 – 1/4; 1/2 – 1/8.

**Following days**

Many days of board work follow this pattern—adding and subtracting, multiplying and dividing fractions, continuing to build up a sense in learners of the relative size of fractions and mixed numbers.
References

